Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Compulsory Part

S5 – S6 Core Assignment Set 1

Name: ______________________ Telephone: ____________

Student number: ______________________ Centre: ____________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings (except for multiple choice questions) must be clearly shown.
3. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S5 – S6 Core
Phase 1 – Lesson 1

1. A

Reduce the graph of $y = \sin x$ along the x-axis to $\frac{1}{2}$ times the original.

Then reduce along y-axis to $\frac{1}{3}$ times the original to get the graph shown.

$$\text{Coordinates of } T = \left(\frac{1}{2} \times 270°,\ \frac{1}{3} \times (-1)\right)$$
$$= \left(135°,\ -\frac{1}{3}\right)$$

2. C

Enlarge to 3 times of the original along the x-axis:

$y = \tan x \quad \longrightarrow \quad y = \tan \frac{x}{3}$

Reduce to $\frac{1}{2}$ times the original along the y-axis:

$y = \tan \frac{x}{3} \quad \longrightarrow \quad y = \frac{1}{2} \tan \frac{x}{3}$

3. B

$y = f(x) \quad \longrightarrow \quad y = f(-x) \quad \longrightarrow \quad y = f(-2x)$

Reflect about y-axis.

Reduce along x-axis to $\frac{1}{2}$ times the original.

The answer is B.

4. A

The graph of exponential curve was translated b units upwards. So, $b > 0$. Exponential graph is increasing if the base is greater than 1. So, $a^{-1} > 1$ and therefore $0 < a < 1$.

5. B

I. ✓. The graph $y = f(x-2)$ is 2 units on the right of $y = f(x)$, which can possibly be $y = g(x)$.

II. ✓. The graph $y = f(-x+2)$ is obtained by translating the graph $y = f(x)$ 2 units to the left [to $y = f(x+2)$]
and then reflect with respect to the y-axis, which is also possible.

III. ✗. The graph $y = f(-x-2)$ is obtained by translating the graph $y = f(x)$ 2 units to the right [to $y = f(x-2)$]
and then reflect with respect to the y-axis. The vertex should lie on the left of the y-axis.

6. D

y-intercept $= 0.5^{0-1} + 1 = 2 + 1 = 3$

When x is large, 0.5^{x-1} is close to 0, and so the value of y is close to 1.

Only option D satisfies these.

7. B

From $y = f(x)$,
reflect about x-axis $\rightarrow$ $y = -f(x)$
translate leftwards by 2 units $\rightarrow$ $y = -f(x+2)$

8. C

From $y = f(x)$ to $y = g(x)$:
Translate leftwards by 4 units and downwards by 3 units.
So, $g(x) = f(x+4) - 3$

9. B

$y = \frac{4}{x} \longrightarrow y = \frac{4}{x+4}$

I. ✓.

II. ✗. When $x = -3.5$, $y = \frac{4}{-3.5+4} = 8$

III. ✓. When $x = 0$, $y = \frac{4}{4} = 1$, only one y-intercept.

10. A

Note that $f(2) = f(5) = 0$.

A. ✓.

B. ✗. When $x = -1$, $y = f(1-1) = f(0) \neq 0$.

C. ✗. When $x = -1$, $y = -f(-1) - 1 \neq 0$.

D. ✗. When $x = -1$, $y = -f(-1) + 1 \neq 0$.

11. C

$g(x) = -\frac{1}{2}f(x)$

The graph of $y = f(x)$ is reduced along the y-axis to $\frac{1}{2}$ times the original and then is reflected about the x-axis to the graph of $y = g(x)$.
The answer is C.

12. A

$y = f(x) \longrightarrow y = -f(x) \longrightarrow y = -f(x) + 1$

The new graph is obtained by

(a) reflect the graph about the x-axis,

(b) translate upwards by 1 unit.

The answer is A.

13. B

$y = f(x) \longrightarrow y = f(-x)$: reflect about y-axis

$y = f(-x) \longrightarrow y = f(-x) - 5$: translate downwards by 5 units

14. B

The graph passes through $(0,\ 1)$ and $(90,\ 0)$.

The answer is B.

15. A

From $y = f(x)$ to $y = f(2x + 1)$:

Translate 1 unit leftwards $\longrightarrow$ reduce along x-axis to $\frac{1}{2}$ the original $\Rightarrow$ Option A

Alternative transformation:

Reduce along x-axis to $\frac{1}{2}$ the original $\longrightarrow$ translate $\frac{1}{2}$ units leftwards $\Rightarrow$ Option A

Alternative solution:

Note that $f(0) = f(5) = 0$.

$g\left(-\frac{1}{2}\right) = f\left[2\left(-\frac{1}{2}\right) + 1\right] = f(0) = 0$ and $g(2) = f[2(2) + 1] = f(5) = 0$.

The graph of $y = g(x)$ has x-intercepts $-\frac{1}{2}$ and 5 $\Rightarrow$ Option A.

16. B

2 units upwards $\Rightarrow y = \log x + 2$

Enlarged along y-axis to 3 times the original $\Rightarrow y = 3(\log x + 2) = 3\log x + 6$

17. C

$y = f(x) \quad \longrightarrow \quad y = -f(x) \quad \longrightarrow \quad y = -f(x - 2)$

Reflect about the x-axis. Translate rightwards by 2 units.

The answer is C.

18. A

The graph $y = -f(x - 1)$ is obtained by

translating the graph of $y = f(x)$ rightwards by 1 unit;

reflect the resulting graph with respect to the x-axis.

Only option A satisfies this.

19. D

Let $f(x) = (x - h)^2 + k$. The resulting graph is

$y = -f(x - h)$

$\quad = -(x - 2h)^2 - k$

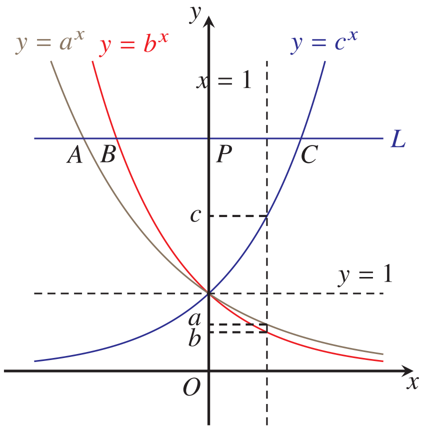
20. B

Draw the line $x = 1$ and $y = 1$.
From the graph, $0 < b < a < 1 < c$.

I. ✓. From the graph, $0 < a < 1$ and $0 < b < 1$.
So, $ab < 1$.

II. ✗. The graph $y = b^x$ and $y = c^x$ is symmetric about the y-axis. Thus, $b^{-1} = c$.
We have $ac > bc = 1$.

III. ✓.



21. D

Check the value of $g(0.5)$.

A. ✗. $g(0.5) = f\left(\frac{0.5}{2}\right) - 1 = f(0.25) - 1$

B. ✗. $g(0.5) = f(1+1) = f(2)$

C. ✗. $g(0.5) = f\left(\frac{0.5}{2} - 1\right) = f(-0.75)$

D. ✓. $g(0.5) = f(1) - 1 = 0 - 1 = -1$

22. D

We have $f(3) = 2$ and $g(3) = 1$.

A. ✗. $g(3) = -2f(3) + 2 = -2(2) + 2 = -2 \neq 1$

B. ✗. $g(3) = -2f(3) + 3 = -2(2) + 3 = -1 \neq 1$

C. ✗. $g(3) = -\frac{1}{2}f(3) + 3 = -1 + 3 = 2 \neq 1$

D. ✓. $g(3) = -\frac{1}{2}f(3) + 2 = -1 + 2 = 1$

23. B

I. ✓. When $x = 0$, $4 = p + q\tan 0° = p$.

II. ✗. $y = \tan x°$ is an increasing curve. Since the curve here is also increasing, $q > 0$.

III. ✓. When $x = -\alpha$, $0 = 4 + q\tan(-\alpha)° \Rightarrow \tan\alpha° = \frac{4}{q} > 0$

24. $\boxed{\text{A}}$

Note that $f(2) = 1$ corresponds to $g(5) = -1$.

A. ✓. $g(5) = f(2) - 2 = 1 - 2 = -1$

B. ✗. $g(5) = f(2) + 2 = 1 + 2 = 3 \neq -1$

C. ✗. $g(5) = f(8) - 2 = ?$

D. ✗. $g(5) = f(8) + 2 = ?$

25. $\boxed{\text{C}}$

It shows the graphs of $y = f(x)$ and $y = -f(x-5)$.

The graph of $y = f(x)$ is reflected about the x-axis and is translated leftwards by 5 units.

26. (a) $\frac{2}{1+i} \times \frac{2}{1-i} = \frac{2}{a}$ 1M

$\frac{4}{1+1} = \frac{2}{a}$

$a = 1$ 1A

$\frac{2}{1+i} + \frac{2}{1-i} = -\frac{b}{1}$

$\frac{(2-2i)+(2+2i)}{1+1} = -b$

$b = -2$ 1A

(b) Let $g(x) = f(x) + k = x^2 - 2x + 2 + k$, where $k \neq 0$.

If $y = g(x)$ has two x-intercepts, then $g(x) = 0$ has two distinct real roots and

$\Delta = 2^2 - 4(1)(2+k) > 0$ 1M

$-4 - 4k > 0$

$k < -1$

The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ downwards by more than 1 unit. 1A

27. (a) $2x^3 + 3x^2 - 5x - 6 = 0$

$(x+1)(2x^2 + x - 6) = 0$ 1M

$(x+1)(x+2)(2x-3) = 0$

$x = -1 \quad \text{or} \quad -2 \quad \text{or} \quad \frac{3}{2}$

The x-intercepts are -1, -2 and $\frac{3}{2}$. 1A

(b) $g(x) = f\left(\frac{x}{2}\right)$ and $h(x) = -g(x)$

$h(x) = -f\left(\frac{x}{2}\right)$ 1M+1A

$= -\frac{x^3}{4} - \frac{3x^2}{4} + \frac{5x}{2} + 6$ 1A

28. (a) $h(x) = f(-x+3)$ 1M

$= 2(-x+3)^2 - 8(-x+3) - 10$

$= 2x^2 - 4x - 16$ 1A

(b) Let $f(x) = h(x+c)$, where c is a constant.

$2x^2 - 8x - 10 = 2(x+c)^2 - 4(x+c) - 16$ 1M

Compare coefficient of x,

$-8 = 4c - 4$

$c = -1$ 1A

The graph of $y = h(x)$ is translated to the right by 1 unit to become the graph of $y = f(x)$. 1A

29. (a) $\log_4 y = -\frac{1}{2}x + 3$ 1A

$y = 4^{-\frac{1}{2}x+3}$ 1M

$= 2^{-x} \cdot 64$

$= 64\left(\frac{1}{2}\right)^x$

So, $k = 64$ and $a = \frac{1}{2}$. 1A

(b) $h(x) = 4f(x)$ 1A

$= 64\left(\frac{1}{2}\right)^x \cdot \left(\frac{1}{2}\right)^{-2}$

$= 64\left(\frac{1}{2}\right)^{x-2}$

$= f(x-2)$

The graph $y = h(x)$ can be obtained by translating the graph of $y = f(x)$ 2 units rightwards.

The claim is agreed. 1A

30. (a) $g(x) = f(-x) = (-x-3)^2 = (x+3)^2$ 1A

(b) $g(x) = f(x+6)$.

The graph can be obtained by translating the graph of $y = f(x)$ leftwards by 6 units. 1M

The claim is agreed. 1A

(c) Vertex of the graph of $y = g(x)$ is at $(-3, -0)$. 1A

Image of the vertex is at $\left(-\frac{3}{2}, 0-5\right) = \left(-\frac{3}{2}, -5\right)$.

Thus, coordinates of vertex of the graph of $y = h(x)$ are $\left(-\frac{3}{2}, -5\right)$. 1A

31. (a) $p(x) = f(x) + 3$ 1A

$q(x) = 2f(x)$ 1A

$r(x) = -f(x)$ 1A

(b) $f(x) = -2[f(x) + 3] = -2f(x) - 6$ 1A

(c) (i) $y = -2[(x^2 - 2x + 3) + 3]$ 1M

$= -2x^2 + 4x - 12$ 1A

(ii) The graph obtained is

$$y = 2[-(x^2 - 2x + 3)] + 3$$

$$= -2x^2 + 4x - 3$$ 1A

Hence, the graph obtained is not the same as the graph in (c)(i). 1

END OF PAPER