Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Compulsory Part

S4 – S5 Core Assignment Set 2

Name: ______________________ Telephone: ______________

Student number: ______________________ Centre: ______________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings (except for multiple choice questions) must be clearly shown.
3. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S4 – S5 Core
Phase 1 – Lesson 2

1. B

$\angle BOD = 53° \times 2 = 106°.$
$\angle BCD = \dfrac{360° - 106°}{2} = 127°.$
$\angle CDO = 360° - 106° - 41° - 127° = 86°.$

2. B

$\angle ADC = 180° - 92° = 88°$

$\angle ADO = 88° - 61° = 27°$

$\angle OAD = \angle ADO = 27°$

$\angle AOD = 180° - 27° - 27° = 126°$

3. C

$\angle PQT = b - a$ and $\angle TUR = \angle PQT = b - a$.

$(b - a) + c = 180°$

$a = b + c - 180°$

4. D

$\angle BDC = \angle BAC = 32°$
$\angle CBD = \angle BDC \times \dfrac{3}{2} = 48°$
$\angle ABC = \angle BDC \times \dfrac{1}{2} = 16°$
$\angle ACD = 180° - 16° - 32° - 48° = 84°$

$\angle AED = 180° - 84° = 96°$

5. A

$OB = OC$. $\angle ODC = \angle ODB = a$.

$\angle BOC = 2\angle BAC = 2b$.

$\angle BOC + \angle BDC = 180°$

$a + b = 90°$

6. D

$\angle CDF = x + 43°$ and $\angle DCF = \angle BAD = x$.

$x + (x + 43°) + 33° = 180°$

$x = 52°$

7. C

$\angle CAB = \angle CDB = 56°.$
$\angle ABC = \angle ACB = \dfrac{180° - 56°}{2} = 62°.$
$\angle ADE = \angle ABC = 62°.$

8. D

Let $\angle ADB = 3\theta$. Then $\angle BDC = 2\theta$ and $\angle CBD = \theta$.

$80° + (3\theta + 2\theta) = 180°$

$\theta = 20°$

$\angle DBA = 80° - \theta = 60°$.

9. A

Note that $\triangle PAC \sim \triangle PDB$.

$$\frac{PA}{PD} = \frac{PC}{PB}$$
$$\frac{4}{5 + CD} = \frac{5}{4 + 5}$$
$$CD = 2.2$$

10. C

$\angle EBC = \angle EDF = 76°$.

$\angle AEC = \angle EBC \times \dfrac{1 + 1}{3 + 1} = 38°$.

$\angle ABC = 180° - \angle AEC = 142°$.

11. C

$\angle ABC = 180° - 68° = 112°$

$\angle ABD = 90°$ and $\angle CBD = 112° - 90° = 22°$

$\angle BDC = \angle CBD = 22°$ and $\angle ADB = 68° - 22° = 46°$

12. A

$\angle BAD = 180° - \angle DCE = 82°$.

$\angle ABE = \angle BAD - \angle AEB = 47°$.

13. C

$\angle BAD : \angle BCD = (3 + 5) : (3 + 4)$

$= 8 : 7$

Since $\angle BAD + \angle BCD = 180°$, $\angle BAD = 180° \times \dfrac{8}{8 + 7} = 96°$.

14. D

Reflex $\angle SOP = 360° - 100° = 260°$

$\angle STP = \dfrac{260°}{2} = 130°$

$\angle RTP = 180° - 135° = 45°$

$\angle STR = 130° - 45° = 85°$

15. C

$\angle AOE = 2 \times 25° = 50°$.

Since $OA = AE$, $\angle AEO = \angle AOE = 50°$.

$\angle ABC = 180° - \angle ADC = 155°$.

$\angle ECB = \angle ABC - \angle BEC = 105°$.

16. D

$\angle P + \angle R = 180°$

$$\angle P = 180° \times \frac{3}{3+5}$$
$$= 67.5°$$

$$\angle Q = \angle P \times \frac{4}{3}$$
$$= 90°$$

$\angle S = 180° - 90° = 90°$

17. C

$\overset{\frown}{AOB} : \overset{\frown}{BOC} : \overset{\frown}{COD} = \overset{\frown}{AB} : \overset{\frown}{BC} : \overset{\frown}{CD} = 4 : 6 : 5$

$$\angle AOB + \angle BOC = (360° - 105°) \times \frac{4+6}{4+6+5}$$
$$= 170°$$

Reflex $\angle AOC = 360° - 170° = 190°$

$$\angle ABC = \frac{190°}{2} = 95°$$

18. B

Note that $\triangle ADE \sim \triangle ACB$.

$$\frac{AE}{AB} = \frac{DE}{CB}$$
$$\frac{1.2}{3.6} = \frac{DE}{3}$$
$$DE = 1\text{ cm}$$

Note that $\triangle BFC \sim \triangle DFE$.

$$\frac{DF}{BF} = \frac{DE}{BC}$$
$$\frac{DF}{2.7} = \frac{1}{3}$$
$$DE = 0.9\text{ cm}$$

19. C

$\angle BCE = 180° - \angle BFE = 65°$

$\angle DCE = 105° - 65° = 40°$

Since $\overset{\frown}{CD} = \overset{\frown}{DE}$, $\angle DEC = \angle DCE = 40°$.

$\angle CEF = 95° - 40° = 55°$

$\angle ABF = \angle CEF = 55°$

$\angle BAF = 115° - 55° = 60°$

20. C

Draw line segment BE.

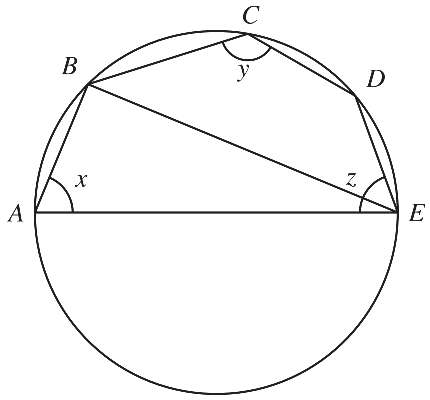
$\angle ABE = 90°$

$x + \angle AEB = 180° - 90° = 90°$

$y + \angle BED = 180°$

$x + y + z = x + (\angle AEB + \angle BED) + y$

$= 270°$



21. $\angle BAD = 180° - 108°$ 1M

$= 72°$

$\angle DCE + 80° + 72° = 180°$ 1M

$\angle DCE = 28°$ 1A

$\angle ADE = \angle DAE = \angle DCE = 28°$ 1M

$\angle ABE = \angle ADE = 28°$ 1M

$\angle EBC + 28° + 108° = 180°$

$\angle EBC = 44°$ 1A

22. $\angle ADB = \angle BDC = 32°$ 1A

$\angle ABD = 90°$ 1A

$\angle BAD = 180° - 90° - 32° = 58°$ 1A

$\angle BCD = 180° - 58° = 122°$ 1M+1A

23. (a) $\angle ADE = 90°$ *(given)*

$\angle ABC = 90°$ *(given)*

$= \angle ADE$

$\angle EAD = \angle CAB$ *(common ∠)*

$\triangle ADE \sim \triangle ABC$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $EB = AE = 20$ cm. 1A

$DE = \sqrt{20^2 - 16^2} = 12$ cm 1M

Since $\triangle ADE \sim \triangle ABC$,

$$\frac{BC}{AB} = \frac{DE}{AD}$$

$$\frac{BC}{20+20} = \frac{12}{16}$$ 1M

$$BC = 30 \text{ cm}$$

Since $\angle CBE = 90°$, CE is a diameter of the required circle. 1M

Let the radius of the circle be r.

$$(2r)^2 - 20^2 = 30^2$$

$$r^2 = 325$$

Required area is $r^2\pi = 325\pi$ cm^2. 1A

24. (a) $\angle QPR = \angle SPT$ (given)

$PR = PS$ (given)

$\angle PTS = \angle PQR$ (equal chords, equal $\angle$s)

$\triangle PQR \cong \triangle PTS$ (AAS)

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\triangle PQR \cong \triangle PTS$

$QR = TS$ 1M

$\angle TRS = \angle SQT$ 1M

So, $QT // RS$. The claim is agreed. 1A

25. (a) Let G be a point on DE such that $AE // BG$.

$\angle ABG + 104° = 180°$ 1M

$\angle ABG = 76°$ 1A

$\angle CBG + 128° = 180°$

$\angle CBG = 52°$

$\angle ABC = 76° + 52° = 128°$ 1A

(b) $\angle ABC = 128°$ (proved)

$\angle DCB = 128°$ (given)

$= \angle ABC$

$BC = CB$ (common side)

$\angle BAC = \angle BDC$ ($\angle$s in the same segment)

$\triangle ABC \cong \triangle DCB$ (AAS)

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

26. (a) $AD = DB$ (given)

$AE = EC$ (given)

$DE // BC$ (midpoint theorem)

$\angle AED = \angle ACB = 60°$ (corr. $\angle$s, $DF // BC$)

$\angle AEF = 180° - \angle AED = 120°$ (adj. $\angle$s on st. line)

$\angle BDF = 180° - \angle ABC$ (int. $\angle$s, $DF // BC$)

$= 120° = \angle AEF$

$\angle EAF = \angle FBC$ ($\angle$s in the same segment)

$\angle DFB = \angle FBC$ (alt. $\angle$s, $DF // BC$)

$= \angle EAF$

$\triangle AEF \sim \triangle FDB$ (AA)

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(b) Let $EF = x$. $\triangle ADE$ is equilateral triangle.

$\triangle AEF \sim \triangle FDB$ (proved)

$$\frac{AE}{EF} = \frac{DF}{BD}$$ 1M

$$\frac{1}{x} = \frac{1+x}{1}$$ 1A

$$0 = x^2 + x - 1$$

$$x = \frac{-1+\sqrt{5}}{2} \quad \text{or} \quad \frac{-1-\sqrt{5}}{2} \text{ (rejected)}$$ 1A

27. (a) $\angle ADC = 90°$ and $\angle CDE = 90° - 60° = 30°$ 1A

$\angle DCE = 180° - 60° = 120°$ 1A

$\angle CED = 180° - 120° - 30° = 30° = \angle CDE$

Thus, $CD = CE$ and $\triangle CDE$ is isosceles. 1A

(b) In $\triangle BEF$, $\angle EBF = 90°$ and $\angle BEF = 30°$.

Thus, $\angle EF = \dfrac{BE}{\cos 30°}$.

In $\triangle CDE$, $DE = 2CE \cos 30°$.

In $\triangle ADF$, $DF = AD = 5\sqrt{3}$ cm.

$$2CE \cos 30° + \frac{BE}{\cos 30°} = 5\sqrt{3}$$ 1M

$$BC \cos 30° + \frac{BC}{2 \cos 30°} = 5\sqrt{3}$$

$$BC = 6 \text{ cm}$$ 1A

(c) Let M be the midpoint of DF.

$GM = \dfrac{DF}{2} \times \tan 30° = \dfrac{5}{2}$ cm 1A

$CE = \dfrac{BC}{2} = 3$ cm

Area of $\triangle GDE = \dfrac{1}{2}(GM)(DE)$

$$\begin{aligned}\text{Area of } \triangle CDE &= \frac{1}{2}(CE \sin 30°)(DE) \\ &= \frac{1}{2}(1.5)(DE) \\ &< \frac{1}{2}(GM)(DE)\end{aligned}$$

Area of $\triangle CDE <$ area of $\triangle GDE$

The claim is agreed. 1A

END OF PAPER