Dexter Wong & His Mathematics Team
Summer Course 2022 – 2023
MATHEMATICS Compulsory Part
S4 – S5 Core Assignment Set 1

Name: ____________________ Telephone: __________

Student number: ____________________ Centre: __________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings (except for multiple choice questions) must be clearly shown.
3. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S4 – S5 Core
Phase 1 – Lesson 1

1. $\boxed{\text{D}}$

$\angle DCK = 90° - 32° = 58°$.
$\angle ABK = \angle DCK = 58°$ and $\angle AKB = \angle ABK = 58°$.
$\angle KDC = \angle BAK = 180° - 58° - 58° = 64°$.

2. $\boxed{\text{B}}$

$\angle ACB = \angle QCB = 90°$.
$\angle BPC = \angle BAC = 30°$.
$\angle ACP = \angle QPC + \angle CQP = 55°$.
$\angle PCB = 90° - 55° = 35°$.

3. $\boxed{\text{D}}$

Note that $\triangle BMC \sim \triangle DMA$.
Let the radius be r.

$$\frac{BM}{DM} = \frac{CM}{AM}$$
$$\frac{1.5r}{3} = \frac{4}{0.5r}$$
$$0.75r^2 = 12$$
$$r = 4 \quad \text{or} \quad -4 \text{ (rejected)}$$

4. $\boxed{\text{B}}$

$\angle CAD = 90°$ and $\angle BAC = 116° - 90° = 26°$
$\angle AEF = 78° - 26° = 52°$
$\angle ACD = \angle AEF = 52°$
$\angle ABD = 52° - 26° = 26°$

5. $\boxed{\text{C}}$

$\angle QPS = 90°$
$\angle QPR = 28° + 32° = 60°$
$\angle RPS = 90° - 60° = 30°$

6. $\boxed{\text{B}}$

$\angle BAO = \angle ABO = 15°$
$\angle CAO = \angle ACO = 40°$
$\angle BOC = 2\angle BAC = 2(15° + 40°) = 110°$

7. $\boxed{\text{B}}$

$\angle DCE = \dfrac{44°}{2} = 22°$.
$\angle BCE + \angle BAE = 180°$

$$(y - 22°) + x = 180°$$
$$x + y = 202°$$

8. $\boxed{\text{C}}$

$\angle ACE = 34° - 20° = 14°$
$\angle ABD = \angle ACE = 14°$
$\angle ADB = x + 46°$
$\angle ACB = \angle ADB = x + 46°$
Consider $\triangle ABC$.

$$(x + 14°) + 34° + (x + 46°) = 180°$$
$$x = 43°$$

9. $\boxed{\text{D}}$

$\angle ADC = 90°$. $\angle ADE = 90° - 48° = 42°$.
$\angle BCE = \angle ADE = 42°$ and $\angle BEC = 180° - 42° - 56° = 82°$.

10. $\boxed{\text{B}}$

$\angle BDC = \dfrac{70°}{2} = 35°$ and $\angle ACD = 90°$.
$\angle BEC = 90° + 35° = 125°$.

11. $\boxed{\text{B}}$

Area of $ABCD$ is the sum of area of $\triangle ACD$ and $\triangle ABC$.
Area of $\triangle ACD$ is fixed while area of $\triangle ABC$ is maximum when B is furthest away from AC.
$\angle ADC = 90°$ and $AC = \sqrt{6^2 + 8^2} = 10\,\text{cm}$.

$$\text{Required area} = \frac{(6)(8)}{2} + \frac{(10)(5)}{2} = 49\,\text{cm}^2$$

12. $\boxed{\text{B}}$

$\angle OCA = \angle OAC = 35°$
$\angle ACO = 180° - 35° - 35° = 110°$
Reflex $\angle ACO = 360° - 110° = 250°$
$\angle ABC = \dfrac{250°}{2} = 125°$
$\angle ACB = \angle OAC = 35°$
$\angle BAC = 180° - 125° - 35° = 20°$

13. $\boxed{\text{D}}$

$\angle CED = \dfrac{30°}{2} = 15°$.
$\angle ABD + \angle AED = 180°$

$$x + (y + 15°) = 180°$$
$$x + y = 165°$$

14. $\boxed{\text{C}}$

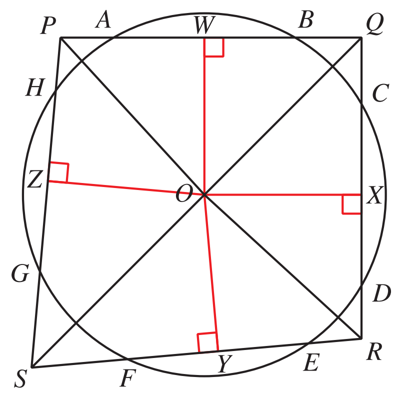
Let W, X, Y and Z be midpoints of AB, CD, EF and GH respectively.

$\triangle PWO \cong \triangle PZO$, $\triangle QWO \cong \triangle QXO$.

$\triangle RXO \cong \triangle RYO$, $\triangle SYO \cong \triangle SZO$.

$\angle XOZ = 2 \times \angle POQ = 172°$.

$\angle ROS = \frac{1}{2}(\text{reflex } \angle XOZ) = \frac{360° - 172°}{2} = 94°$.



15. $\boxed{\text{B}}$

$\angle DAO = \angle ADO = 46°$ and $\angle OCA = \angle OAC = 64° - 46° = 18°$.

$\angle AOC = 180° - 2 \times 18° = 144°$ and reflex $\angle AOC = 360° - 144° = 216°$.

$\angle ABC = \frac{1}{2} \times 216° = 108°$.

16. $\boxed{\text{B}}$

$\angle ABE = \angle BAD + \angle ADB = 33° + 48° = 81°$

$\angle ACE = \angle ABE = 81°$

$\angle ACB = 90°$

$\angle BCE = 90° - 81° = 9°$

17. $\boxed{\text{C}}$

$\angle ABE = \angle ADE = 28°$ and $\angle ABC = 90°$.

So, $\angle CBE = 28° + 90° = 118°$.

18. $\boxed{\text{B}}$

$\angle EBD = \angle ADB - \angle DEB = 40°$ and $\angle CAD = \angle CBD = 40°$.

$\angle BAD = 50° + 40° = 90°$.

So, BD is a diameter of the circle.

$\angle ACB = \angle ADB = 55°$ and $\angle ABC = 180° - 50° - 55° = 75° \neq 90°$.

So, AC is not a diameter of the circle.

19. $\boxed{\text{A}}$

$\angle BAD = 180° - \angle ABC = 38°$. $\angle BOD = 2 \times 38° = 76°$

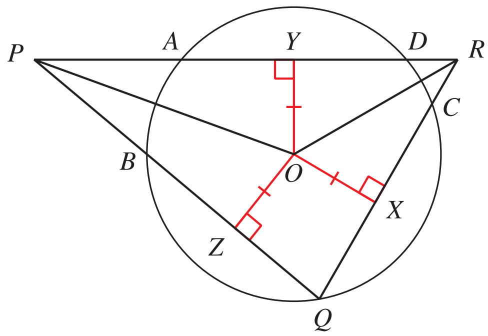
$\angle ADC = \angle ABC = 142°$. $\angle ODE = 180° - 142° = 38°$.

$\angle BED = 76° + 38° = 114°$

20. $\boxed{\text{D}}$

Since $AD = BQ = CQ$, the distances from three chords to centre are equal.

Note that $\triangle OYP \cong \triangle OZP$ and $\triangle OYR \cong \triangle OXR$.
Let $\angle YPO = \angle ZPO = a$ and $\angle YRO = \angle XRO = b$. In $\triangle POR$,



$$130° + a + b = 180°$$
$$a + b = 50°$$

In $\triangle PQR$,

$$\angle PQR + 2(a + b) = 180°$$
$$\angle PQR = 80°$$

21. $\boxed{\text{D}}$

$\angle CAB = \angle CDB = \angle DBA = \angle DCA = 20°$ and $\angle ACB = 90°$.
$\angle CBD = 180° - 20° - 90° - 20° = 50°$.

22. $\boxed{\text{A}}$

Let $\angle ODC = x$.
$\angle BOA = 180° - 2 \times 66° = 48°$.
$\angle COD = \angle ODC = x$ and $\angle OBC = \angle OCB = \angle COD + \angle CDO = 2x$.
$\angle BOA = \angle OBD + \angle ODB$

$$48° = 2x + x$$
$$x = 16°$$

23. $\boxed{\text{D}}$

Reflex $\angle AOC = 2x$. So, $y = 360° - 2x$.

24. $\boxed{\text{A}}$

Let $OA = r$ cm.
$\angle OMA = 90°$
Consider $\triangle OAM$.

$$r^2 = (r-2)^2 + 3^2$$
$$r^2 = r^2 - 4r + 13$$
$$r = 3.25$$

$OM = r - 2 = 1.25$ cm

25. $\boxed{\text{B}}$

Let $\angle PRM = x$.

Since $ON = NR$, we have $\angle NOR = x$.

$\angle QMR = \dfrac{\angle NOR}{2} = \dfrac{x}{2}$

In $\triangle MQR$,

$$x + \frac{x}{2} = 36^\circ$$
$$x = 24^\circ$$

26. $\boxed{\text{D}}$

$OM \perp AB$, $ON \perp BC$ and $AB = BC$. So, $OM = ON$.

$\angle MON = 360^\circ - 90^\circ \times 2 - 68^\circ = 112^\circ$.

$\angle OMN = \dfrac{180^\circ - 112^\circ}{2} = 34^\circ$.

27. $\boxed{\text{C}}$

$\angle ABE = 90^\circ$ and $\angle AEB = \angle ACB = 30^\circ$

$\angle EAB = 180^\circ - 90^\circ - 30^\circ = 60^\circ$

28. $\boxed{\text{D}}$

Draw three lines joining the midpoints of chord and the centre as shown.

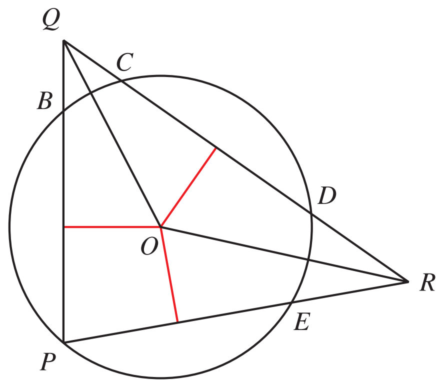
Since $PB = PE = CD$, lengths of three drawn line segments are equal.

There are some congruent triangles such that OQ and OR are angle bisectors of $\angle PQR$ and $\angle QRP$ respectively.

Let $\angle OQR = x$ and $\angle QRO = y$.

$$2x + 2y + 68^\circ = 180^\circ$$
$$x + y = 56^\circ$$

$\angle QOR = 180^\circ - x - y = 124^\circ$



29. $\boxed{\text{A}}$

Note that $\angle ACB = 90^\circ$ and AB is a diameter of the circle.

$$\text{Required area} = \frac{1}{2} \times \left(\frac{25}{2}\right)^2 \pi - \frac{(7)(24)}{2}$$
$$\approx 161 \text{ cm}^2$$

30. $\boxed{\text{B}}$

$\angle BCA = \angle BDA = d$ and $\angle ECD = \angle EBD = b$.

$\angle BCD = b + c + d = 90^\circ$.

31. $\boxed{\text{D}}$

$\angle ADB = 90°$

$\angle ABD = 180° - 90° - 72° = 18°$

$\angle ODB = \angle ABD = 18°$

$\angle ODC = \angle DCE = 46°$

$\angle EDC = \angle ODC - \angle ODB = 46° - 18° = 28°$

32. $\boxed{\text{B}}$

Let O be the centre of the circle, and N be the midpoint of AB such that $ON \perp AB$.

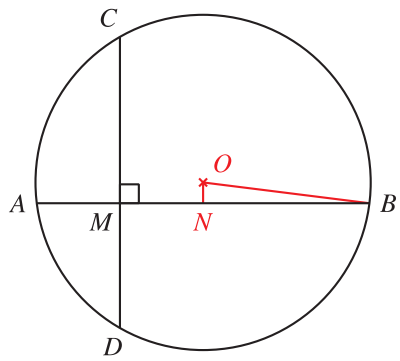
$\triangle AMD \sim \triangle CMB$

$\frac{AM}{MD} = \frac{CM}{MB}$

$CM = 4$

$NB = \frac{2+6}{2} = 4$ and $ON = 4 - \frac{4+3}{2} = 0.5$

Radius $= \sqrt{4^2 + 0.5^2} \approx 4.03$



33. $\boxed{\text{C}}$

Let M, N and T be midpoints of AB, CD and EF respectively.

$AB = CD = EF \quad \Rightarrow \quad OT = OM = ON$

$\triangle OMQ \cong \triangle ONQ$ and $\triangle ONR \cong \triangle OTR$

Let $\angle OQN = a$ and $\angle ORN = b$.

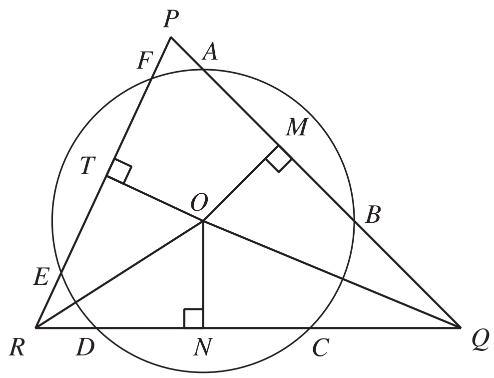
We have $\angle OQM = a$ and $ORT = b$.

$2a + 2b + 42° = 180°$

$a + b = 69°$

$\angle QOR = 180° - (a+b)$

$= 111°$



34. $\boxed{\text{B}}$

Let the centre of circle be O. Radius $= \frac{10}{2} = 5\text{ cm}$.

$\angle BOC = 2 \times 35° = 70°$ and $\angle AOB = 110°$

Required area $= 5^2\pi \times \frac{110°}{360°} - \frac{1}{2}(5)^2 \sin 110°$

$\approx 12.3\text{ cm}^2$

35. $\boxed{\text{A}}$

$OM \perp BC$ and $OM = \sqrt{OC^2 - MC^2} = 8$.

$AM = \sqrt{OA^2 - OM^2} = 4\sqrt{5}$ and $AD = 2AM = 8\sqrt{5}$.

36. $\boxed{\text{B}}$

$\angle BCD = 90°$ and $\angle BDC = \angle DBC = \dfrac{180° - 90°}{2} = 45°$.

$\angle BAC = \angle BDC = 45°$ and $\angle ABE = \angle AEB = \dfrac{180° - 45°}{2} = 67.5°$.

$\angle BAD = 90°$ and $\angle BDA = 180° - 90° - 67.5° = 22.5°$.

$\angle ACB = \angle BDA = 22.5°$.

37. $\boxed{\text{C}}$

$AB = 2AM = 8$ cm.

$OM \perp AB$, $ON \perp CD$ and $OM = ON$. So, $CD = AB = 8$ cm.

38. $\boxed{\text{B}}$

$\angle CDB = x$ and $\angle DBE = x - 35°$.

$(x - 35°) + x = 81°$

$x = 58°$

39. $\angle ABC = 90°$ 1A

$\angle CBD = \angle DAC = 25°$ 1A

$\angle ABP = 90° - 25° = 65°$ 1A

$\angle BAP = 180° - 65° - 75° = 40°$ 1A

40. $\angle BAC = \angle BDC = 50°$ 1A

$\angle ABD = \dfrac{180° - 50° - 20°}{2} = 55°$ 1M+1A

$\theta = \angle ACD = \angle ABD = 55°$ 1A

41. $\angle DAC = \angle CBD = 25°$ 1A

$\angle ADC = 90°$ 1A

$\angle ACD = 180° - 90° - 25° = 65°$ 1A

$\angle CDE = 128° - 65° = 63°$ 1A

42. (a) Let $\angle AOE = x$.

Then $\angle AEO = x$ and $\angle DBE = \dfrac{x}{2}$. 1A

$x + \dfrac{x}{2} = 48°$

$x = 32°$ 1A

(b) $\angle OAB = 32° + 32° = 64°$ and $\angle AOB = 180° - 2 \times 64° = 52°$. 1A

$\dfrac{\overset{\frown}{AB}}{AE} = \dfrac{2(AO)\pi \times \frac{52°}{360°}}{AO}$ 1M

$\approx 0.908 < 1$

So, $\overset{\frown}{AB} < AE$. The claim is agreed. 1A

43. (a) $\angle DBA = \angle ACD = x$ 1A

$\angle CAB = \angle CEB + \angle ACD = x + 15°$ 1A

(b) $\angle ACB = 90°$ 1A

$(x + 15°) + 90° + (45° + x) = 180°$ 1M

$x = 15°$ 1A

44. Since $AF = BF$, $\angle FAB = \angle ABF = 35°$. 1A

$\angle BFC = 35° + 35° = 70°$

Since $BE // CD$, $\angle GCD = \angle BFC = 70°$. 1A

$\angle BDC = \angle BAC = 35°$ 1A

Consider $\triangle CDG$, $\angle BGC = 35° + 70° = 105°$. 1A

45. (a) $AB^2 + BC^2 - AC^2 = 5^2 + 12^2 - 13^2 = 0$ 1M

So, $AB^2 + BC^2 = AC^2$ and $\angle ABC = 90°$. 1A

(b) $\angle ADC = 180° - \angle ABC = 90°$

Let $AD = DC = x$ cm.

$$x^2 + x^2 = 13^2$$ 1M

$$x = \frac{13\sqrt{2}}{2}$$

Required perimeter $= 5 + 12 + \frac{13\sqrt{2}}{2} \times 2$ 1M

$= (17 + 13\sqrt{2})$ cm 1A

(c) Let h be the shortest distance from D to AC.

$$\frac{1}{2}(h)(13) = \frac{1}{2}\left(\frac{13\sqrt{2}}{2}\right)^2$$ 1M

$$h = 6.5$$

There is no such point F. 1A

46. (a) $\triangle ADE \sim \triangle BCE$ 1A

$$\frac{CE}{DE} = \frac{BE}{AE}$$ 1M

$$\frac{CE}{14} = \frac{62.5}{50}$$

$CE = 17.5$ cm 1A

(b) (i) In $\triangle BCE$,

$$BC^2 + CE^2 = 60^2 + 17.5^2$$ 1M

$$= 62.5^2 = BE^2$$

$\angle BCE = 90°$ *(converse of Pyth. theorem)* 1M

Thus, AB is a diameter of the circle. *(converse of $\angle$ in semi-circle).* 1A

(ii) $AB = \sqrt{60^2 + (50 + 17.5)^2} = \frac{15\sqrt{145}}{2}$ 1M

Let θ be the angle at centre respect to $\overset{\frown}{BC}$.

$$\tan\frac{\theta}{2} = \tan\angle BAC$$

$$= \frac{60}{67.5}$$ 1M

$$\theta \approx 83.267\,078\,67°$$

$\overset{\frown}{BC} = \frac{15\sqrt{145}}{2}\pi\left(\frac{\theta}{360°}\right) \approx 65.6$ cm 1A

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