

Dexter Wong & His Mathematics Team

暑期課程 2022 – 2023

數學 延伸部分

S5 – S6 M2 積分 功課 第 1 套

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上課日：日 / 一 / 二 / 三 / 四 / 五 / 六

考生須知

1. 本試卷各題均須作答，答案須寫在本試題答題簿中預留的空位內。
2. 除特別指明外，須詳細列出所有算式。
3. 除特別指明外，數值答案須用真確值表示。
4. 本試卷的附圖不一定依比例繪成。

建議題解



派發於暑期課程
S5 – S6 M2 積分
第 1 期 – 第 1 堂

1. $\int \frac{3 + 7\sqrt{t} - \sqrt[3]{t}}{\sqrt[3]{t}} dt = \int (3t^{-\frac{1}{3}} + 7t^{\frac{1}{6}} - 1) dt$ 1M
 $= \frac{9}{2}t^{\frac{2}{3}} + 6t^{\frac{7}{6}} - t + \text{常數}$ 1A

2. (a) $\frac{1}{x} - \frac{1}{x+5} = \frac{(x+5) - x}{x(x+5)} = \frac{5}{x^2 + 5x}$ 1A
 (b) $\int \frac{dx}{x^2 + 5x} = \frac{1}{5} \int \left(\frac{1}{x} - \frac{1}{x+5} \right) dx$ 1M
 $= \frac{\ln|x|}{5} - \frac{\ln|x+5|}{5} + \text{常數}$ 1A
 $= \frac{1}{5} \ln \left| \frac{x}{x+5} \right| + \text{常數}$

3. $\int \cos^4 2x dx = \frac{1}{4} \int (1 + \cos 4x)^2 dx$ 1M
 $= \frac{1}{4} \int (1 + 2\cos 4x + \cos^2 4x) dx$ 1M
 $= \frac{1}{4} \int \left[1 + 2\cos 4x + \frac{1}{2}(1 + \cos 8x) \right] dx$ 1M
 $= \frac{3x}{8} + \frac{\sin 4x}{8} + \frac{\sin 8x}{64} + \text{常數}$ 1A

4. $\int \frac{4x+9}{8x^2+38x+45} dx = \int \frac{1}{2x+5} dx$ 1M
 $= \frac{\ln|2x+5|}{2} + \text{常數}$ 1A

5. $\int (\cos 2x - \sin 6x)^2 dx = \int (\cos^2 2x - 2\cos 2x \sin 6x + \sin^2 6x) dx$ 1M
 $= \int \left[\frac{1}{2}(\cos 4x + 1) - (\sin 8x + \sin 4x) + \frac{1}{2}(1 - \cos 12x) \right] dx$ 1M
 $= \frac{\sin 4x}{8} + x + \frac{\cos 8x}{8} + \frac{\cos 4x}{4} - \frac{\sin 12x}{24} + \text{常數}$ 1A

6. $\int \sin 8x \sin 5x \sin x dx = \frac{1}{2} \int \sin x (\cos 3x - \cos 13x) dx$ 1M
 $= \frac{1}{4} \int [(\sin 4x - \sin 2x) - (\sin 14x - \sin 12x)] dx$ 1M
 $= -\frac{\cos 4x}{16} + \frac{\cos 2x}{8} + \frac{\cos 14x}{56} - \frac{\cos 12x}{48} + \text{常數}$ 1A

7. (a) $P + \frac{Q}{x+4} + \frac{R}{2x-5} = \frac{P(x+4)(2x-5) + Q(2x-5) + R(x+4)}{(x+4)(2x-5)}$
 $= \frac{2Px^2 + (3P+2Q+R)x + (-20P-5Q+4R)}{(x+4)(2x-5)}$
 故此， $2P = 4$ 、 $3P + 2Q + R = 11$ 及 $-20P - 5Q + 4R = -59$ 。 1M
 求解後，可得 $P = 2$ 、 $Q = 3$ 及 $R = -1$ 。 1A+1A
 (b) $\int \frac{4x^2 + 11x - 59}{2x^2 + 3x - 20} dx = \int \left(2 + \frac{3}{x+4} - \frac{1}{2x-5} \right) dx$ 1M
 $= 2x + 3 \ln|x+4| - \frac{\ln|2x-5|}{2} + \text{常數}$ 1A

$$8. \int \cos 3x \sin x \sin 4x \, dx = \frac{1}{2} \int \sin x (\sin 7x + \sin x) \, dx \quad 1M$$

$$= \frac{1}{4} \int [(\cos 6x - \cos 8x) + (1 - \cos 2x)] \, dx \quad 1M$$

$$= \frac{x}{4} - \frac{\sin 8x}{32} + \frac{\sin 6x}{24} - \frac{\sin 2x}{8} + \text{常數} \quad 1A$$

$$9. \int \frac{2u^8 + 4u^2 - 10}{u^3} \, du = \int (2u^5 + 4u^{-1} - 10u^{-3}) \, du \quad 1M$$

$$= \frac{u^6}{3} + 4 \ln |u| + 5u^{-2} + \text{常數} \quad 1A$$

$$10. (a) \frac{dy}{dx} = e^x + xe^x \quad 1A$$

$$(b) \text{ By (a), } xe^x = \frac{dy}{dx} - e^x$$

$$\int xe^x \, dx = \int \left(\frac{dy}{dx} - e^x \right) dx \quad 1M$$

$$= y - e^x + \text{常數} \quad 1A$$

$$= xe^x - e^x + \text{常數}$$

$$11. \int \frac{x+2}{\sqrt[3]{x} + \sqrt[3]{2}} \, dx = \int (x^{\frac{2}{3}} - 2^{\frac{1}{3}}x^{\frac{1}{3}} + 2^{\frac{2}{3}}) \, dx \quad 1M$$

$$= \frac{3x^{\frac{5}{3}}}{5} - \frac{3\sqrt[3]{2}x^{\frac{4}{3}}}{4} + 2^{\frac{2}{3}}x + \text{常數} \quad 1A$$

$$12. \int \frac{x+8}{x+6} \, dx = \int \left(1 + \frac{2}{x+6} \right) dx \quad 1M$$

$$= x + 2 \ln |x+6| + \text{常數} \quad 1A$$

$$13. \int \sin^2 x \cos^2 x \, dx = \int \left(\frac{\sin 2x}{2} \right)^2 dx \quad 1M$$

$$= \frac{1}{4} \int \frac{1}{2} (1 - \cos 4x) \, dx \quad 1M$$

$$= \frac{x}{8} - \frac{\sin 4x}{32} + \text{常數} \quad 1A$$

$$14. \int \sin \frac{7\theta}{2} \sin \frac{\theta}{2} \, d\theta = \frac{1}{2} \int (\cos 3\theta - \cos 4\theta) \, d\theta \quad 1M$$

$$= \frac{\sin 3\theta}{6} - \frac{\sin 4\theta}{8} + \text{常數} \quad 1A$$

$$15. \int (6^{x+1} - 5x^6) \, dx = \int (e^{(x+1) \ln 6} - 5x^6) \, dx \quad 1M$$

$$= \frac{e^{(x+1) \ln 6}}{\ln 6} - \frac{5x^7}{7} + \text{常數}$$

$$= \frac{6^{x+1}}{\ln 6} - \frac{5x^7}{7} + \text{常數} \quad 1A$$

16. $\int \frac{\sin^2 t + 2 \cos^3 t}{\cos^2 t} dt = \int (\tan^2 t + 2 \cos t) dt$ 1M
 $= \int (\sec^2 t - 1 + 2 \cos t) dt$ 1M
 $= \tan t - t + 2 \sin t + \text{常數}$ 1A
17. (a) $\frac{Px + Q}{x^2 - 2} + \frac{R}{x + 3} = \frac{(Px + Q)(x + 3) + R(x^2 - 2)}{(x^2 - 2)(x + 3)}$
 $= \frac{(P + R)x^2 + (3P + Q)x + (3Q - 2R)}{(x^2 - 2)(x + 3)}$
 故此， $P + R = 6$ 、 $3P + Q = 3$ 及 $3Q - 2R = -10$. 1M
 求解後，可得 $P = 1$ 、 $Q = 0$ 及 $R = 5$ 。 1A+1A
- (b) $\int \frac{6x^2 + 3x - 10}{x^3 + 3x^2 - 2x - 6} dx = \int \frac{6x^2 + 3x - 10}{(x^2 - 2)(x + 3)} dx$
 $= \int \left(\frac{x}{x^2 - 2} + \frac{5}{x + 3} \right) dx$
 設 $u = x^2 - 2$ 。則 $du = 2x dx$ 。 1M
- $\int \frac{6x^2 + 3x - 10}{x^3 + 3x^2 - 2x - 6} dx = \frac{1}{2} \int \frac{du}{u} + \int \frac{5}{x + 3} dx$ 1M
 $= \frac{\ln |x^2 - 2|}{2} + 5 \ln |x + 3| + \text{常數}$ 1A
18. (a) $\frac{1}{x - 2} - \frac{1}{x + 2} = \frac{(x + 2) - (x - 2)}{(x - 2)(x + 2)}$
 $= \frac{4}{x^2 - 4}$ 1
- (b) $\int \frac{dx}{x^2 - 4} = \frac{1}{4} \int \left(\frac{1}{x - 2} - \frac{1}{x + 2} \right) dx$ 1M
 $= \frac{\ln |x - 2|}{4} - \frac{\ln |x + 2|}{4} + \text{常數}$ 1A
19. $\int (e^x + 1)^3 dx = \int (e^{3x} + 3e^{2x} + 3e^x + 1) dx$ 1M
 $= \frac{e^{3x}}{3} + \frac{3e^{2x}}{2} + 3e^x + x + \text{常數}$ 1A
20. $\int (1 - 2 \sin^2 4x) dx = \int \cos 8x dx$ 1M
 $= \frac{\sin 8x}{8} + \text{常數}$ 1A
21. $\int \frac{dx}{\sqrt{2 - 2x} - \sqrt{4 - 2x}} = \int \frac{\sqrt{2 - 2x} + \sqrt{4 - 2x}}{(2 - 2x) - (4 - 2x)} dx$ 1M
 $= -\frac{1}{2} \int (\sqrt{2 - 2x} + \sqrt{4 - 2x}) dx$
 $= \frac{(2 - 2x)^{\frac{3}{2}}}{6} + \frac{(4 - 2x)^{\frac{3}{2}}}{6} + \text{常數}$ 1A

$$\begin{aligned}
 22. \int 9^x \cdot 2^x \, dx &= \int 18^x \, dx \\
 &= \int e^{x \ln 18} \, dx && 1M \\
 &= \frac{e^{x \ln 18}}{\ln 18} + \text{常數} \\
 &= \frac{18^x}{\ln 18} + \text{常數} && 1A
 \end{aligned}$$